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Global estimation of Hidden Markov models using interval arithmetic

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Resumo

Hidden Markov Models are important tools in statistics and applied mathematics, with applications in speech recognition, physics, mathematical finance and biology. The Hidden Markov Models we consider are formed by two discrete time and finite state stochastic process. The first process is a Markov chain and is not observable directly. Instead, we observe a second process which is driven by the hidden process. In order to extract conclusions from a Hidden Markov Model we must estimate the parameters defining it. In this article we present global optimization techniques to estimate these parameters by maximum likelihood and compare our estimates with the ones obtained by the local likelihood maximization methods already described in the literature. In order to evaluate the global maximum we provide an interval branch and bound algorithm based on interval Newton method and a symmetry breaking scheme. The algorithm starts with a local Baum-Welch method, which provides a warm lower bound for the problem. We also derive KKT conditions to obtain a new box elimination test. Our algorithm is able, in a successful execution, to find a box with prescribed width which rigorously contains at least one feasible point for the problem and such that the solution is an epsilon-global maximum. The objective function for this problem can be evaluated by the so called backward and forward recursions. In fact we can use only one of these recursions or we can combine both to evaluate function and its derivatives. These three formulations are equivalent using exact arithmetic. However they will usually be different in interval arithmetic due to the lack the distributivity law. In order to accelerate the convergence of upper bound of the global maximum we implement and compare interval extensions for the forward, backward and forward-backward equations and their respective derivatives. In order to make our interval bounds tight, we consider enclosures based on Taylor expansion of first and second orders and centered forms. We handle the underflow problems which arise frequently in the estimation problem for Hidden Markov models introducing a new scaling scheme which is not based on taking the log of the objective function. We present the results of numerical experiments illustrating the effectiveness of our approach.